Bank of Canada Banque du Canada

Working Paper 2003-24 / Document de travail 2003-24

Forecasting and Analyzing World Commodity Prices

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ISSN 1192-5434

Printed in Canada on recycled paper

Bank of Canada Working Paper 2003-24

August 2003

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The views expressed in this paper are those of the authors.
No responsibility for them should be attributed to the Bank of Canada.

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Acknowledgements

The authors would like to thank seminar participants at the Bank of Canada for useful comments. Special thanks also to Donald Coletti, Allan Crawford, Simon van Norden, and Jaime Marquez. An earlier version of this paper was presented at the VII Meeting of the Central Banks Research Network of the Americas, hosted by the Central Bank of Guatemala, 22–24 November 2002.

103-06698/

Abstract

The authors develop simple econometric models to analyze and forecast two components of the Bank of Canada commodity price index: the Bank of Canada non-energy (BCNE) commodity prices and the West Texas Intermediate crude oil price. They present different methodologies to identify transitory and permanent components of movements in these prices. A structural vector autoregressive model is used for real BCNE prices and a multiple structural-break technique is employed for real crude oil prices. The authors use these transitory and permanent components to develop forecasting models. They assess various aspects of the models' performance. Their main results indicate that: (i) the world economic activity and real U.S.-dollar effective exchange rate explain much of the cyclical variation of real BCNE prices, (ii) real crude oil prices have two structural breaks over the sample period, and recently their link with the world economic activity has been quite strong, and (iii) the models outperform benchmark models, namely a vector autoregressive model, an autoregressive model, and a random-walk model, in terms of out-of-sample forecasting.

JEL classification: C5
Bank classification: Econometric and statistical methods

Résumé

Les auteurs élaborent des modèles économétriques simples en vue d'analyser et de prévoir l'évolution de deux composantes de l'indice des prix des produits de base de la Banque du Canada, à savoir les prix des produits de base non énergétiques et ceux du pétrole brut West Texas Intermediate. Ils décrivent diverses méthodologies qui permettent de distinguer les éléments transitoires et permanents des fluctuations de ces prix. Les auteurs utilisent un modèle vectoriel autorégressif structurel dans le cas des prix réels des produits de base non énergétiques, et une technique de détection de points de rupture structurels multiples pour ceux du brut. À l'aide des éléments transitoires et permanents ainsi départagés, ils construisent des modèles de prévision, dont ils évaluent ensuite la qualité sous différents angles. Les principales conclusions qui se dégagent de l'étude sont les suivantes : i) les variations cycliques des prix réels des produits de base non énergétiques sont largement attribuables à l'évolution de l'économie mondiale et à celle du taux de change effectif réel du dollar américain; ii) les prix réels du pétrole brut présentent deux points de rupture structurels au cours de la période d'estimation et, dans la dernière partie de cette période, ils sont très étroitement liés à l'activité économique mondiale; iii) en dehors de la période d'estimation, les modèles proposés par les auteurs ont un pouvoir de prévision supérieur à celui des modèles de référence (soit un modèle vectoriel autorégressif, un modèle autorégressif et une marche aléatoire).

Classification JEL : C5
Classification de la Banque : Méthodes économétriques et statistiques

1. Introduction

The resource sector has traditionally played an important role in the Canadian economy, especially in the area of foreign trade. Over the past decade, total exports of commodities have represented, on average, about 41 per cent of Canada's exports of goods[1] and 15 per cent of Canada's gross domestic product (GDP). Consequently, changes in world commodity prices have historically been a key determinant of Canada's terms of trade, which in turn have affected the real income of Canadians.

Bank staff have designed the Bank of Canada commodity price index (BCPI) to track the prices paid for key Canadian commodities. The BCPI is a fixed-weighted index of the spot or transaction prices of 23 commodities produced in Canada and sold in world markets.[2] All components of the BCPI are priced in U.S. dollars. The choice of commodities is determined by their importance in Canadian production, subject to limitations imposed by data availability. In this paper, we focus on two components of the BCPI: Bank of Canada non-energy (BCNE) commodity prices and the West Texas Intermediate (WTI) crude oil price. To obtain real commodity prices, we divide by the U.S. GDP deflator.

We employ two different empirical approaches to model commodity prices. For real BCNE prices, we combine a structural vector autoregressive (SVAR) model with a single-equation model. The SVAR gives us a historical decomposition of movements in real BCNE prices, and projects the permanent (or long-run) component of prices, while the transitory (or short-run) component of real BCNE prices is forecasted with a single equation model. We find that this approach successfully captures the strong linkage of real BCNE prices with the world economic activity and the real U.S.-dollar effective exchange rate in the short run.[3] A 1 per cent positive shock to world economic activity leads to an approximately 6 per cent peak response of real BCNE prices, while the response to the real U.S.-dollar effective exchange rate shock is small but it is statistically significant and exhibits the expected sign.[4] We also find that the variance of the transitory component of real BCNE prices accounts for approximately 60 per cent of total real

1. This ratio is defined as the share of nominal commodity exports in total nominal exports over the period 1990–2001.
2. Appendix A describes the BCPI and its components. The weight of each commodity in the index is based on the average value of Canadian production of the commodity over the 1982–90 period.
3. The U.S.-dollar effective exchange rate is defined as a U.S.-export weighted average of the exchange rate of the U.S. dollar relative to the currencies of Japan (17.59 per cent), the United Kingdom (8.22 per cent), Mexico (14.52 per cent), Canada (35.40 per cent), and the euro zone (24.27 per cent).
4. We use the world output gap as a proxy for overall world economic activity, where the output gap is generated using the Hodrick-Prescott (HP) filter.

BCNE price variance. This result is consistent with numerous other studies of commodity prices (for example, Borensztein and Reinhart 1994). In terms of out-of-sample forecasting, our approach outperforms a VAR model and an autoregressive (AR) model.

For real crude oil prices, we use a statistical multiple structural-break approach to identify significant shifts in OPEC behaviour. After controlling for these mean shifts, we find a very strong role for world economic activity in the determination of oil prices. We estimate that a 1 per cent positive shock to world economic activity leads to an approximately 12 per cent peak response of real crude oil prices with a lag of 2 to 3 quarters. In terms of out-of-sample forecasts, the real oil-price model outperforms an AR model and a random-walk (RW) model.

This paper is organized as follows. Section 2 briefly reviews recent literature on commodity prices. In section 3, we present the methodology as well as results for the real BCNE price model. The methodologies and results for the real crude-oil-price model are described in section 4. Section 5 concludes.

2. Literature Review

This section briefly reviews some of the recent economic literature pertaining to price formation in world commodity markets. A common theme in many of these studies has been an attempt to disentangle commodity price movements into a cyclical and a long-term movement. This distinction is important for forecasting commodity prices both in the short run and long run.

Various methodologies have been used to disentangle the trend movement of world commodity prices from the cycle. Reinhart and Wickham (1994) apply two different approaches, namely the Beveridge and Nelson (1981) technique and the Harvey (1985) approach. The first approach is a pure reduced-form time-series technique used for the decomposition of a time-series variable, and the second is a structural time-series approach using the Kalman filter. Each of the two approaches has its own strengths and weaknesses. The pure mechanical filters can easily split a time series into cyclical and permanent components, but they lack economic fundamentals. Although the Kalman filter contains certain economic information, it often does not perform very well in practice if the assumptions of normal distributions for disturbances and the initial state vector are violated. When the normality assumption is dropped, there is no longer a guarantee that the Kalman filter will give the conditional mean of the state vector; i.e., the estimates of the state vector could be conditionally biased (see Harvey 1989 for details). Moreover, it becomes more cumbersome to calculate the likelihood function without the normality assumption.

Following a study by Reinhart and Wickham (1994), Borensztein and Reinhart (1994) adopt a structural model to identify the key fundamentals behind commodity prices, and more importantly to quantify the relative contributions of demand and supply shocks. On the demand side, they find that the real U.S.-dollar effective exchange rate and the state of the business cycle in industrial countries are closely linked to the cyclical movement of world commodity prices. On the supply side, strong productivity growth of commodity sectors relative to the rest of the economy and the increased commodity supply relative to the rest of the economy are the primary causes of the downward trend of commodity prices. Using a variance decomposition, the authors conclude that both types of shocks contribute to the total variation of commodity prices in the near term and that around 60 per cent of the variation is caused by demand shocks.

Cashin, Liang, and McDermott (2000) examine the persistence of shocks to commodity prices. They use a median-unbiased estimation procedure proposed by Andrews (1993), instead of a unit root test, to check the persistence of shocks. Using International Monetary Fund (IMF) data on 60 individual commodity prices, they find that shocks to most commodity prices are long-lasting (reflected by the high value of the half-life of a unit shock), and the variability of the persistence is fairly large. Cashin and McDermott (2001) use much longer sample periods and examine whether the long-run behaviour of commodity prices has changed. In particular, they look at the trend of most commodity prices, the duration of price booms and slumps, and the volatility of price movements. They apply various statistical tests and compare the patterns of commodity price movements across different sample periods. The authors conclude that there has been an apparent downward trend in real commodity prices over the last 140 years because of relative productivity growth in commodity sectors and a structural change in supply conditions.[5] Moreover, the short-term volatility is highly related to the business cycle.

In practice, numerous methodologies have been employed to disentangle transitory and permanent movements in commodity prices. Though convenient to apply, pure time-series filters suffer from a lack of structural economic fundamentals. In contrast, although structural models are constructed based on economic theory, they are often costly and time-consuming to develop and maintain. For instance, it would be very costly to develop and maintain models for 23 individual components of the BCPI. Therefore, as a compromise, we combine basic time-series approaches with simple economic theory to develop econometric models for two components of the Bank of Canada commodity price index.

5. Coletti (1992) examines a small set of non-energy commodities that include mainly industrial materials (e.g., metals, minerals, and forest products) over the 1900–91 period. He finds no obvious secular decline in the relative prices of those commodities.

3. The Real BCNE Price Model

This section consists of three subsections. The first two describe the methodology used to identify and forecast the transitory and permanent components of real BCNE prices. The third describes the results.

3.1 Identifying the transitory and permanent components of real BCNE prices

Figure 1 plots the evolution of real BCNE prices over the sample period. It is evident that the non-energy commodity price index has experienced a downward trend, consistent with the earlier studies on primary commodity prices. This characteristic could be largely explained by the rise in relative productivity in commodity-producing sectors, as well as the increase in supply conditions of most primary commodities over the past three decades. We also notice that the series exhibits substantial variations around trend over the course of business cycles. We use an SVAR approach to decompose historical BCNE prices into transitory and permanent components. Under this approach, a number of economic restrictions are imposed on the long-run effects of different types of shocks. The main strength of the SVAR methodology is that one does not have to impose a fully specified theoretical structure, and the data are allowed to speak. The only assumptions are that the variable of interest (real BCNE prices) can be decomposed into one or more permanent components and one or more transitory components, and that the transitory shocks are uncorrelated with the permanent shocks. The SVAR methodology, however, has its own weakness. Notably, the results are often sensitive to the choice of variables included in the estimation. Also, results can be affected by the number of lags chosen in the reduced form, assumptions on the order of integration of variables, and the presence of cointegrating relationships among variables.

In our model, variable selection is based on economic theory and the findings of previous studies. To capture the information about transitory shifts in real BCNE prices arising from changes in world economic conditions, we use the G-7 output gap as a proxy for world economic activity.[6] The G-7 inflation rate,[7] a proxy for the global inflation rate, is added to capture the importance of

6. The G-7 output gap is generated using the SVAR methodology for the United States (see Lalonde 1998) and the HP filter for the rest of the G-7 countries. We take the sum of individual output gaps weighted by each country's share in the composition of the G-7 output evaluated at purchasing-power parity. We use the term "world output gap" through the rest of the paper.

7. The G-7 inflation rate is generated by taking the sum of individual inflation rates weighted by each country's share in the composition of the G-7 output evaluated at purchasing-power parity. We use the term "global inflation rate" through the rest of the paper.

having a nominal anchor in the model, as suggested by the SVAR literature.[8] In light of the empirical studies on world commodity prices noted in section 2, we include two additional demand indicators—the real U.S. long-term interest rate as a proxy for the real world interest rate, and the real U.S.-dollar effective exchange rate—to identify the cyclical component of real BCNE prices.[9] In addition, we have attempted to include some supply-side determinants of the permanent component of real BCNE prices. Given that the real BCNE price is an aggregate price index, however, it is hard to find a proper measure of productivity.[10]

The final SVAR contains the following five variables: real BCNE prices (*Rbcne*), the world output gap (*Wygap*), the global inflation rate ($W\pi$), the real U.S. long-term interest rate (*RRus*), and the real U.S.-dollar effective exchange rate (*Erus*). We assume that the real U.S. interest rate is stationary in levels.[11] ADF tests show that the world output gap is stationary in levels and that the rest of the variables are first-difference stationary. Furthermore, a Johansen cointegration test shows that there is no cointegrating relationship between *Rbcne*, $W\pi$, and *Erus*. Appendix B describes the technical details of the SVAR methodology. We estimate the model over the period 1972–2001 using quarterly data.[12]

3.2 Forecasting the transitory and permanent component of real BCNE prices

The second step of the approach consists of finding the best way to produce forecasts of both the temporary and permanent components that are not only tractable but consistent with projections of the rest of the world economy. We use the VAR to forecast the permanent component of real

8. If monetary policy has a neutral effect across different sectors of the economy, in both the short and long run, the presence of the global inflation rate in the model may not be important. However, because monetary policy may not affect all sectors in the same manner in the short run, it can have a transitory effect on relative prices. Consequently, using real BCNE prices alone may not be sufficient to purge the effects of monetary policy. Out-of-sample forecasting performance of the model that includes the global inflation rate is slightly better than the one that excludes it. Furthermore, results show that real BCNE prices do react, in the short run, to a shock that affects the trend inflation rate.
9. Since world commodities are all priced in U.S. dollars, movements in the real U.S.-dollar effective exchange rate will affect the demand for commodities by countries other than the United States. This in turn will affect prices.
10. If the productivity growth happens only in a particular sector, this tends to lower production costs in this sector relative to the rest of the economy. Consequently, this causes lower prices of goods produced in this sector relative to the aggregate level (i.e., lower relative prices).
11. The augmented Dickey-Fuller (ADF) test provides ambiguous evidence regarding the stationarity of the real U.S. long-term interest rate. However, we assume that it is stationary. The results are robust to this assumption.
12. We estimate the same model over the period 1972–95 and we find that the transitory component of real BCNE prices is almost identical to the one estimated over the full sample period.

BCNE prices and develop a single-equation model to forecast the transitory component. This single equation links the transitory component of real BCNE prices to the world output gap as well as the real U.S. dollar effective exchange rate gap.[13] This equation is defined as:

Regression equation:

$$Rbcnegap_t = A(L)Rbcnegap_{t-1} + B(L)Wygap_t + C(L)Ergap_t ,^{14} \quad (1)$$

where *Rbcnegap* is the transitory component of real BCNE prices (i.e., the real BCNE price minus the SVAR estimates of its permanent component), *Wygap* is the world output gap, and *Ergap* is the real U.S.-dollar effective exchange rate gap. This equation has the advantage of relying on a small number of estimated parameters, which helps to reduce out-of-sample forecasting errors. In addition, it clearly quantifies the impact of the world output gap and the real U.S.-dollar exchange rate gap on the change in the transitory component of real BCNE prices.

3.3 Results of the real BCNE price model

This section describes results of the real BCNE price model. First, we use variance decomposition to quantify the relative importance of supply and demand shocks. Second, we discuss the link between the world output gap and the transitory component of real BCNE prices. Third, we evaluate the forecasting performance of the model.

3.3.1 The relative importance of supply and demand shocks

Table 1 reports the variance decomposition of real BCNE prices at different time horizons. After the first year (step 4), the transitory shocks (i.e., demand shocks) explain almost 60 per cent of the total variance of real BCNE prices. After two years (step 8), however, the contribution of the demand shocks falls dramatically and accounts for approximately 10 per cent of the total. The model shows a significant contribution of demand shocks to real BCNE prices in the short term, and this is consistent with the studies noted earlier. Figure 2 plots the corresponding impulse responses of real BCNE prices to a positive one-standard-deviation *total* transitory and permanent shock.[15]

13. We use the HP filter to generate the real U.S.-dollar exchange rate gap.
14. The real U.S. interest rate is excluded from the equation due primarily to its strong collinearity with the world output gap, but it can still indirectly affect the forecast of real BCNE prices, via its impact on the forecast of the world output gap (i.e., through the forecast of the U.S. output gap).
15. The total transitory shock is defined as the aggregation of four individual transitory shocks presented in the SVAR model. See Appendix A for details.

Real BCNE prices exhibit a small hump-shaped response to the total transitory shock, whereas the response to the permanent shock appears to be more gradual.

3.3.2 The world output gap and the transitory component of real BCNE prices

Figure 3 plots the evolution of both the world output gap and the transitory component of real BCNE prices over the historical period.[16] There is a strong positive relationship between the two variables. The world output gap tracks most of the important cyclical movements of real BCNE prices since the mid-1970s.

Table 2 reports the parameter estimates of equation (1). The Hausman test fails to reject the null hypothesis of exogeneity of the world output gap, and hence we use instrumental variable estimation (IVE). The instruments used for the estimation are four lags of the world output gap, the transitory component of real BCNE prices, the real U.S.-dollar exchange rate gap, and the real U.S. long-term interest rate. The standard errors of the estimated parameters are modified using an eight-lag Newey-West correction. We start with eight lags for each regressor, and then remove the most insignificant estimates one by one until all the remaining coefficients are statistically significant at the 5 per cent significance level. All the coefficient estimates have the expected signs and are statistically significant in the final model. The transitory component of real BCNE prices itself is fairly persistent, with a root of about 0.71, and both the world output gap and the real U.S.-dollar effective exchange rate gap contribute significantly to transitory movements of real BCNE prices. Furthermore, we calculate the relative contributions of a positive one-standard-deviation shock to each regressor in our model to the total response of the real BCNE transitory component. We find that around 80 per cent of the total response comes from shocks to the world output gap (72 per cent) and the real U.S.-dollar exchange rate gap (8 per cent). In other words, only a small fraction (20 per cent) of the response is left unexplained by our model.

Figure 4 plots the real BCNE prices vis-à-vis their permanent component over history. The implied cyclical movements of real BCNE prices are consistent with our priors. Figures 5 and 6 show the responses of real BCNE prices to a 1 per cent positive shock to the world output gap and the real U.S.-dollar effective exchange rate gap, respectively. The peak response of real BCNE prices to the world output gap shock is about 6 per cent and it occurs almost contemporaneously with the peak of the world output gap itself. In comparison, the response to a shock to the real

16. As a robustness check, we also try the U.S. output gap. We find that models with the world output gap outperform those with the U.S. output gap in most cases. All results for the U.S. output gap model are available upon request.

U.S.-dollar effective exchange rate gap is much smaller, with a peak of about -0.35 per cent, but it exhibits the expected sign.

3.3.3 Out-of-sample forecast of real BCNE prices

World commodity price shocks have a peak impact on the core CPI inflation rate with a lag of 2 to 4 quarters in the Canadian economic projection model used at the Bank. Since monetary policy tends to have its full impact on inflation with a lag of 6 to 8 quarters, the monetary authority will be most interested in forecasts of world commodity prices 2 to 4 quarters ahead.

We evaluate our model's out-of-sample forecasting performance by comparing it with forecasts from two benchmark models: the VAR model and the AR(1) model. Our model forecasts of real BCNE prices combine SVAR forecasts of the permanent component and single-equation forecasts of the transitory component. As stated earlier, we focus on the forecasting horizon that is of interest to the monetary authority, namely 2 to 4 quarters ahead. According to the root-mean-squared error (RMSE) of out-of-sample forecasts from 1992Q1 to 2001Q4,[17] the combined approach uniformly outperforms the two benchmark models regardless of the forecasting horizons (see Table 3) according to smaller values of RMSE.[18]

Tables 4 to 6 report the *p*-values of the forecast-encompassing test statistic, which was originally devised by Chong and Hendry (1986) to compare two competing models based on the out-of-sample forecasting errors. We compare our approach with two benchmark cases: the AR(1) model and the RW model.[19] The encompassing-test results support the use of the combined approach. The results indicate that we cannot reject the null hypothesis of "A encompasses B," which implies that forecasts from either of two benchmark models (model B) are unlikely to improve the forecasting performance of the combined approach (model A) for any forecasting horizon. On the other hand, we can always reject the null hypothesis of "B encompasses A" at the 5 per cent significance level, except for one case, when we compare 4-quarter ahead forecasts with the RW

17. We use a rolling-sample regression to generate out-of-sample forecasts for a given time horizon.
18. The fact that the combined approach outperforms the VAR model could be explained by the following arguments. First, the choice of the variables included in the SVAR were not made on the basis of their ability to forecast real BCNE prices, but on their ability to give information pertinent to the identification of the permanent and transitory components of real BCNE prices. Second, the SVAR literature shows that it is important to include a large number of lags in the SVAR, to identify properly the transitory component of a variable. With a small sample, this strategy is clearly not optimal in terms of out-of-sample forecast performance, because it relies on many estimated parameters. The combined model attempts to address those issues.
19. Given that the results in Table 3 have shown that the AR(1) model performs better than the VAR model, we do not need to report the encompassing test results between our approach and the VAR model.

model. This implies that our combined approach improves the forecasting performance of two benchmark models.

4. The Real Crude-Oil-Price Model

This section consists of two subsections. The first describes the methodology that we use to identify and forecast the transitory and permanent components of real WTI crude oil prices. The second describes the main results of the model.

4.1 Methodologies

Figure 7 plots the series of real crude oil prices over the sample period. As the figure shows, crude oil prices have experienced a few large permanent shifts over history, most notably in 1979–80 and 1985–86.[20] To test for structural breaks in the data (under the assumption that the time and the number of breaks are unknown), we use the methodology proposed by Bai and Perron (1998) (hereafter BP).[21] The strength of the BP methodology is that we can estimate the time and the number of structural breaks endogenously, with allowance for varying parameters across regimes. Given that world oil prices are spot prices, we also allow the model to capture the contemporaneous effect of the world output gap on prices.

4.2 Results of the real crude-oil-price model

This section is divided into three parts. First, we examine the estimation results for the model. Second, we use the estimated model to identify transitory and permanent components of crude oil prices. Third, we evaluate the model's out-of-sample forecasting performance.

4.2.1 Estimation of the real WTI crude oil model

We first consider real crude oil prices over the whole sample period. An ADF test cannot reject the presence of a unit root. When allowing for structural changes, however, we can reject the hypothesis that the series has a unit root.[22]

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20. These large movements in price are related to specific developments in the market, particularly to changes in the behaviour of the OPEC cartel.
21. See Appendix C for a brief discussion of the BP methodology.
22. The sum of AR coefficients is 0.58 and the *t*-statistic is -5.3, compared with a 2.5 per cent critical value of -5.3 (see Zivot and Andrews 1992).

Using the procedure proposed by BP, we estimate a single-equation model as equation (2), with allowance for up to three structural changes. The sample period is from 1974Q2 to 2001Q4. In this framework, all model parameters are allowed to shift at the structural break point. At the 5 per cent level, the test detects two breaks, in 1979Q3 and 1985Q4.[23] These two break points match well two large oil-price shocks over the period. The first oil-price shock in the late 1970s began with the Iranian revolution and the accompanying disruption of its petroleum exports. The outbreak of war between Iran and Iraq in 1980 shook the oil market as well. The second oil-price shock in the mid-1980s was primarily caused by the collapse of OPEC discipline. From 1982 to 1985, OPEC attempted to set production quotas low enough to stabilize prices. These attempts met with repeated failure as various members of OPEC produced beyond their quotas. During most of this period, Saudi Arabia acted as the swing producer, cutting its production to stem the free-falling prices. In late 1985, Saudi Arabia ceased that role and increased its production, which eventually caused the oil-price plunge in 1986. The first experiment we perform is to use three dummy variables to capture the three BP regimes separated by two breaks:

Regression equation

$$Rwti_t = D1^* Dumm1 + D2^*Dumm2 + D3^*Dumm3 + C(L)Rwti_{t-1} + DD(L)Wygap_t + E(L)Ergap_t. \qquad (2)$$

Table 7 reports the ordinary least squares (OLS) estimation results of equation (2) without allowing for varying coefficients across regimes. The purpose of this experiment is to examine the impact of world economic conditions on crude oil prices over the full sample. We report the model parameter estimates for the two cases with and without the real U.S.-dollar exchange rate gap. As Table 7 shows, the results for both cases are almost identical. Real crude oil prices are fairly persistent over the full sample, with an AR root of 0.67 (the sum of the two autoregressive coefficients) and the estimated coefficient associated with the lagged world output gap is about 0.02. The real U.S.-dollar effective exchange rate gap is not statistically significant over the whole sample period.

Tables 8 to 10 report the BP procedure results for three regimes, with allowance for varying coefficients. As the tables show, the estimate of the lagged world output gap changes considerably across regimes. Although it is not statistically significant in the first regime, the estimate has the correct sign. In the second regime, it exhibits the wrong sign, but it is statistically insignificant. In contrast, its magnitude increases substantially in the third and most recent regime, with a value of

23. The test statistics for the sup$F(1|0)$ and sup$F(2|1)$ are 30.7 and 27.4, respectively, compared with the 5 per cent critical values of 20.1 and 22.1.

about 6 per cent, which is almost three times the average value over the entire sample reported in Table 7.[24]

Furthermore, since WTI crude oil prices are spot prices, we would expect crude oil prices to respond immediately to the world output-gap shock. The Hausman test indicates that the null hypothesis of exogeneity of the world output gap is rejected at the 5 per cent level of significance. This implies that applying the OLS to the BP procedure cannot produce consistent estimates, and we should instead use the IVE. The instruments used are four lags of all explanatory variables in the model. Given the small number of observations in the first two regimes, however, applying IVE to the BP procedure tends to give us very biased results.[25] Hence, we use IVE to re-estimate equation (2) only for the third regime, which has a relatively sufficient number of observations.[26] Table 11 reports the IVE of all parameters. The world output gap remains statistically significant at the 5 per cent level of significance, and the estimated coefficient associated with the world output gap is around 4.5 per cent. In our final model, we also add the change in crude oil inventories, another key indicator of crude oil prices. As shown in Table 11, the third lag of the change in crude oil inventories is statistically significant and exhibits the expected sign.[27]

4.2.2 The transitory and permanent components of real crude oil prices

Given the nature of the model, the permanent component consists of three different means caused by two structural breaks. Figure 8 plots the world output gap vis-à-vis the transitory component of real WTI crude oil prices across the three regimes. The closest link between the two variables appears in the most recent regime. Figure 9 shows that a 1 per cent positive shock to the world output gap leads to an approximate 12 per cent peak response of real WTI crude oil prices, with a lag of 2 to 3 quarters.

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24. The real U.S.-dollar effective exchange rate gap is not statistically significant, however; excluding it increases the magnitude and improves the significance of the estimated elasticity of the real crude oil price with respect to the world output gap. Table 11 reports the result of an alternative model excluding the real U.S. exchange rate gap.
25. The bias problem becomes severe in a small sample. See Davidson and Mackinnon (1993) for details.
26. Because Tables 8 to 10 have shown that the strongest link between the world output gap and real crude oil prices is in the third regime, we are more interested in estimating model parameters for this regime. Also, it is the current regime and therefore it has particular relevance for current forecasts.
27. Because only the third lag of the change of crude oil inventory enters our model, we do not need a model to forecast this variable in order to forecast real oil prices over very short time horizons.

4.2.3 Out-of-sample forecast of the real crude-oil-price model

We use the estimated single equation model from Table 11 to forecast real crude oil prices. We compare the two- to four-step-ahead forecasting performance of our model with two benchmark models from 1992Q1 to 2001Q4. Table 12 compares the RMSE of two- to four-step-ahead out-of-sample forecasts of our model with two benchmark models: the RW model and the AR(1) model. It is evident that, regardless of the forecasting horizons concerned, our model uniformly outperforms the other two models, as reflected by the smaller RMSE values.

We also estimate an alternative specification excluding the oil inventory measure. We find that models that exclude the inventory measure always perform worse than those that include it. Furthermore, for near-term forecasting (2 quarters ahead), they are even worse than naive forecasts using an RW model.

Tables 13 to 15 report the *p*-values of forecast-encompassing test statistics. The test results support the use of our model. They show that we cannot reject the null hypothesis of "A encompasses B," which implies that it is impossible to improve the forecasting capability of our model (model A) with the help of forecasts from either of two benchmark models (model B) for any forecast horizon. On the other hand, we can always reject the null hypothesis of "B encompasses A" at the 10 per cent significance level, which implies that our model can provide useful information to improve the forecasting performance of two benchmark models.

5. Conclusion

The variance decomposition has shown that about 60 per cent of the total variation in real BCNE prices is attributed to demand shocks. This is consistent with other studies of non-energy commodity prices in the literature. We have also found a very close link between world economic conditions and transitory movements in real BCNE prices. For real WTI crude oil prices, a multiple structural-break test has identified two structural breaks over the sample period. We used the exogenous mean shifts of real WTI crude oil prices across three different regimes as a measure of the permanent component of real crude oil prices. The corresponding forecasting model shows that the strongest link between the cyclical component of real WTI crude oil prices and world economic conditions occurs in the most recent regime.

In terms of forecasting performance, we have compared two- to four-step-ahead forecasts of our models with benchmark models: a VAR model, an AR(1) model, and an RW model. These tests have shown that our models uniformly outperform the baseline models.

All results in this paper suggest that we can provide better short-term forecasts of world commodity prices relative to benchmark models. However, several extensions to this paper may be worth pursuing. A potential avenue for future work is to put more effort into developing the supply side of the model, and to explore key supply indicators, such as productivity growth in commodity-producing sectors, that can reasonably explain the long-term behaviour of world commodity prices. The models with richer structures can be further developed to better analyze and forecast the commodity prices in both the short and long run.

Bibliography

Andrews, D.W.K. 1993. "Tests for Parameter Instability and Structural Changes with Unknown Change Points." *Econometrica* 61: 821–25.

Bai, J. and P. Perron. 1998. "Computation and Analysis of Multiple Structural-Change Models." *Econometrica* 66: 47–78.

Beveridge, S. and C. Nelson. 1981. "A New Approach to Decomposition of Economic Time Series into Permanent and Transitory Components with Particular Attention to the Measurement of the Business Cycle." *Journal of Monetary Economics* 7: 151–74.

Blanchard, O.J. and D. Quah. 1989. "The Dynamic Effect of Aggregate Demand and Supply Disturbances." *American Economic Review* 79: 655–73.

Borensztein, E. and M. Reinhart. 1994. "The Macroeconomic Determinants of Commodity Prices." International Monetary Fund, Staff Paper No. 41: 236–61.

Cashin, P., H. Liang, and C. McDermott. 2000. "How Persistent Are Shocks to World Commodity Prices?" International Monetary Fund, Working Paper No. 02/47.

Cashin, P. and C. McDermott. 2001. "The Long-Run Behavior of Commodity Prices: Small Trends and Big Variability." International Monetary Fund, Working Paper No. 01/68.

Chong, Y. and D. Hendry. 1986. "Econometric Evaluation of Linear Macroeconomic Models." *Review of Economic Studies* 53: 671–90.

Coletti, D. 1992. "The Long-Run Behaviour of Key Canadian Non-Energy Commodity Prices (1900 to 1991)." *Bank of Canada Review (*Winter 1992–1993): 47–56.

Cuddington, J. and L. Hong. 2000. "Will the Emergence of the Euro Affect World Commodity Prices?" International Monetary Fund, Working Paper No. 00/208.

Davidson, R. and J. MacKinnon. 1993. *Estimation and Inference on Econometrics*. New York: Oxford University Press.

Harvey, A.C. 1985. "Trends and Cycles in Macroeconomic Time Series." *Journal of Business & Economic Statistics* 3: 216–27.

———. 1989. *Forecasting, Structural Time Series Models and the Kalman Filter*. New York: Cambridge University Press.

Hunt, B. 1995. "The Effect of Foreign Demand Shocks on the Canadian Economy: An Analysis using QPM." *Bank of Canada Review* (Autumn): 23–32.

Lalonde, R. 1998. "Le PIB potentiel des États-Unis et ses déterminants: la productivité de la main-d'oeuvre et le taux d'activité." Bank of Canada Working Paper No. 98-13.

———. 2000. "Le modèle U.S.M d'analyse et de projection de l'économie américaine." Bank of Canada Working Paper No. 2000-19.

Newey, W.L. 1985. "Generalized Method of Moments Specification Testing." *Journal of Econometrics* 29: 229–56.

Quah, D. 1992. "The Relative Importance of Permanent and Transitory Components: Identification and Some Theoretical Bounds." *Econometrica* 60: 107–18.

Reinhart, C.M. 1991. "Fiscal Policy, the Real Exchange Rate, and Commodity Prices." International Monetary Fund, Staff Paper No. 38: 506–24.

Reinhart, C.M. and P. Wickham. 1994. "Commodity Prices: Cyclical Weakness or Secular Decline?" International Monetary Fund, Staff Paper 41: 175–213.

St-Amant, P. and S. van Norden. 1997. *Measurement of the Output Gap: A Discussion of Recent Research at the Bank of Canada.* Technical Report No. 79. Ottawa: Bank of Canada.

Zivot, E. and D.W.K. Andrews. 1992. "Further Evidence on the Great Crash, the Oil-Price Shock, and the Unit-Root Hypothesis." *Journal of Business and Statistics* 10: 251–70.

Table 1: Variance Decomposition of Real BCNE Prices (per cent)

Step	Supply shock	Demand shock
1	29	71
4	41	59
8	89	11
16	90	10
∞	100	0

Table 2: Forecasting Equation of the Transitory Component of Real BCNE Prices (*Rbcnegap*) (IVE of equation (1))

Variable	Coefficient
$Rbcnegap_{t-1}$	0.711 (15.79)
$Wygap_t$	0.059 (7.34)
$Wygap_{t-1}$	-0.041 (-5.24)
$Ergap_t$	-0.002 (-2.54)

Note: t-statistics are in parentheses.

Table 3: Out-of-Sample Forecasts of Real BCNE Prices (forecasting period: 1992Q1–2001Q4)

Forecasting horizon	VAR	AR(1) model	Combined approach
	RMSE	RMSE	RMSE
2	0.0633	0.0649	0.0530
3	0.0737	0.0671	0.0564
4	0.0815	0.0748	0.0640

Table 4: Forecast-Encompassing Tests (two-step-ahead forecasts) (*p*-value)

Encompassing tests for A = combined approach and B = benchmark models		
Null hypothesis	**Combined approach vs. AR(1) model**	**Combined approach vs. RW model**
A encompasses B	0.288	0.994
B encompasses A	0.009	0.015

Table 5: Forecast-Encompassing Tests (three-step-ahead forecasts) (*p*-value)

Encompassing tests for A = combined approach and B = benchmark models		
Null hypothesis	**Combined approach vs. AR(1) model**	**Combined approach vs. RW model**
A encompasses B	0.685	0.597
B encompasses A	0.008	0.039

Table 6: Forecast-Encompassing Tests (four-step-ahead forecasts)
(*p*-value)

Encompassing tests for A = combined approach and B = benchmark models		
Null hypothesis	**Combined approach vs. AR(1) model**	**Combined approach vs. RW model**
A encompasses B	0.727	0.781
B encompasses A	0.025	0.061

Table 7: The Real Crude-Oil-Price Model (full sample: 1974Q2–2001Q4)
(OLS estimation of equation (2) with three dummy variables)

Variable	Model 1 (with real U.S.-dollar effective exchange rate gap)	Model 2 (without real U.S.-dollar effective exchange rate gap)
Dumm1	1.013 (6.00)	0.976 (5.81)
Dumm2	1.188 (6.04)	1.144 (5.85)
Dumm3	0.927 (5.94)	0.898 (5.76)
$Rwti_{t-1}$	0.912 (9.57)	0.935 (9.88)
$Rwti_{t-2}$	-0.245 (-2.77)	-0.255 (-2.89)
$Wygap_{t-1}$	0.017 (1.69)	0.023 (2.42)
$Ergap_{t-2}$	-0.005 (-1.48)	-
$\bar{R}^2$	0.85	0.83

Note: *t*-statistics are in parentheses.

Table 8: The Real Crude-Oil-Price Model (Regime 1: 1974Q2–1979Q3)

Variable	Coefficient
Dumm1	1.986 (17.15)
$Rwti_{t-1}$	0.170 (1.97)
$Rwti_{t-2}$	0.170 (2.10)
$Wygap_{t-1}$	0.007 (1.41)
$Ergap_{t-2}$	-0.186 (-1.23)
$\bar{R}^2$	0.92

Note: *t*-statistics are in parentheses.

Table 9: The Real Crude-Oil-Price Model (Regime 2: 1979Q4–1985Q4)

Variable	Coefficient
Dumm2	1.048 (5.50)
$Rwti_{t-1}$	0.868 (7.34)
$Rwti_{t-2}$	-0.165 (-1.61)
$Wygap_{t-1}$	-0.007 (-0.74)
$Ergap_{t-2}$	-0.778 (-1.89)
$\bar{R}^2$	0.92

Note: *t*-statistics are in parentheses.

Table 10: The Real Crude-Oil-Price Model (Regime 3: 1986Q1–2001Q4)

Variable	Coefficient
Dumm2	1.103 (4.01)
$Rwti_{t-1}$	0.873 (8.96)
$Rwti_{t-2}$	-0.261 (-3.74)
$Wygap_{t-1}$	0.055 (2.64)
$Ergap_{t-2}$	-0.390 (0.61)
$\bar{R}^2$	0.92

Note: *t*-statistics are in parentheses.

Table 11: The Real Crude-Oil-Price Model (Regime 3: 1986Q1–2001Q4)
(IVE of regime 3 with contemporaneous world output gap and oil inventories)

Variable	Coefficient
Dumm3	1.071 (3.98)
$Rwti_{t-1}$	0.965 (11.31)
$Rwti_{t-2}$	-0.344 (-3.26)
$Wygap_t$	0.045 (3.07)
$Inventory_{t-3}$	-0.035 (-4.34)

Note: *t*-statistics are in parentheses.

Table 12: The Real Crude-Oil-Price Model: RMSE of Out-of-Sample Forecasts (1992Q1–2001Q4)

Forecasting horizon (quarters)	Our model	RW model	AR(1) model
2	0.134	0.176	0.208
3	0.170	0.225	0.239
4	0.202	0.276	0.246

Table 13: Forecast-Encompassing Tests (two-step-ahead forecasts) (*p*-value)

Encompassing tests for A = our model and B = benchmark models		
Null hypothesis	**Our model vs. AR(1) model**	**Our model vs. RW model**
A encompasses B	0.157	0.499
B encompasses A	0.016	0.033

Table 14: Forecast-Encompassing Tests (three-step-ahead forecasts) (*p*-value)

Encompassing tests for A = our model and B = benchmark models		
Null hypothesis	**Our model vs. AR(1) model**	**Our model vs. RW model**
A encompasses B	0.356	0.654
B encompasses A	0.041	0.086

Table 15: Forecast-Encompassing Tests (four-step-ahead forecasts)
(p-value)

Encompassing tests for A = our model and B = the benchmark models		
Null hypothesis	**Our model vs. AR(1) model**	**Our model vs. RW model**
A encompasses B	0.660	0.411
B encompasses A	0.046	0.077

Figure 1: Real BCNE Prices

Figure 2: Impulse Response of Real BCNE Prices

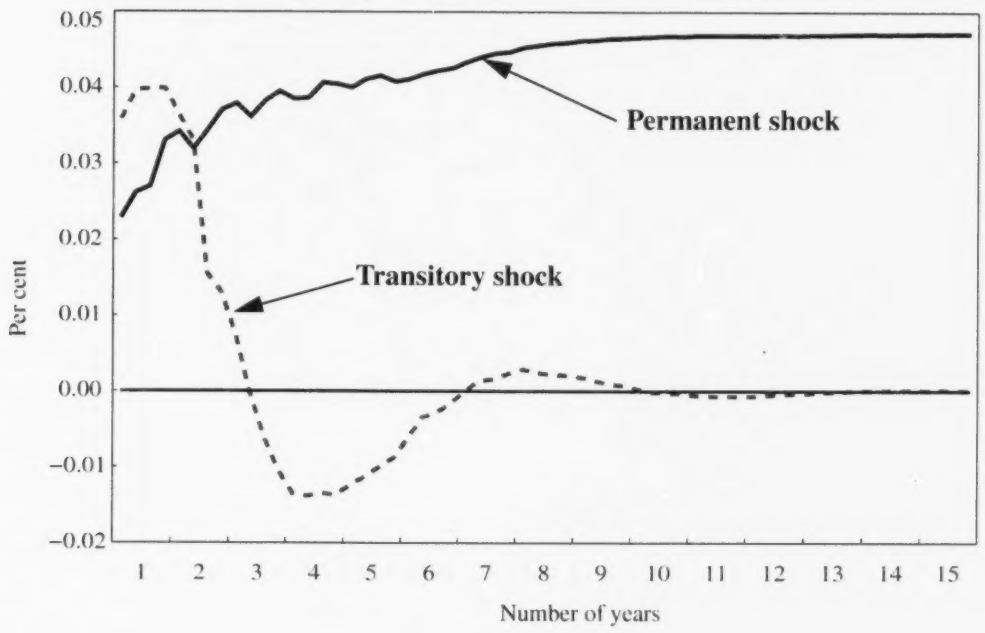


Figure 3: World Output Gap and the Transitory Component of Real BCNE Prices

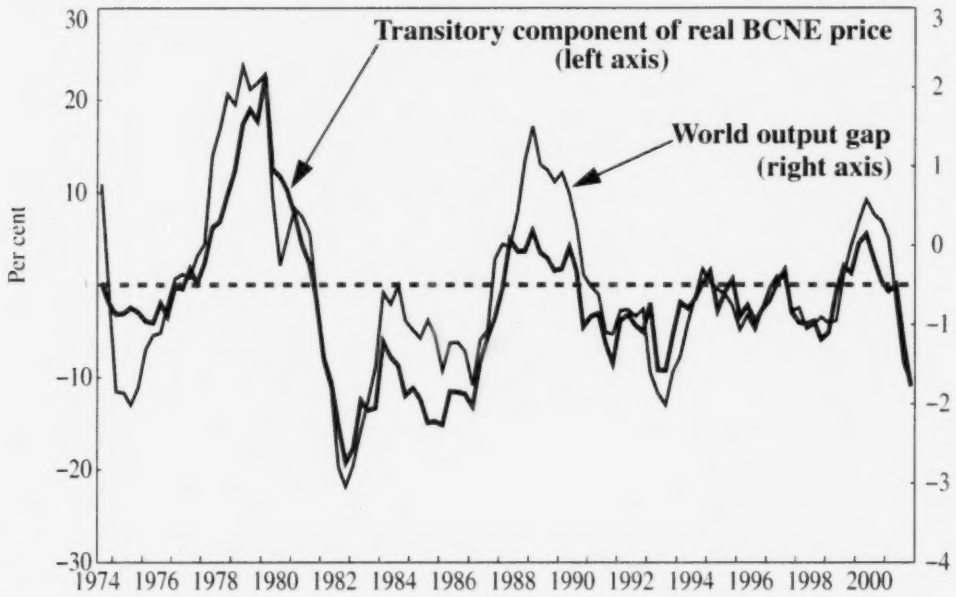


Figure 4: Real BCNE Prices and Their Permanent Component

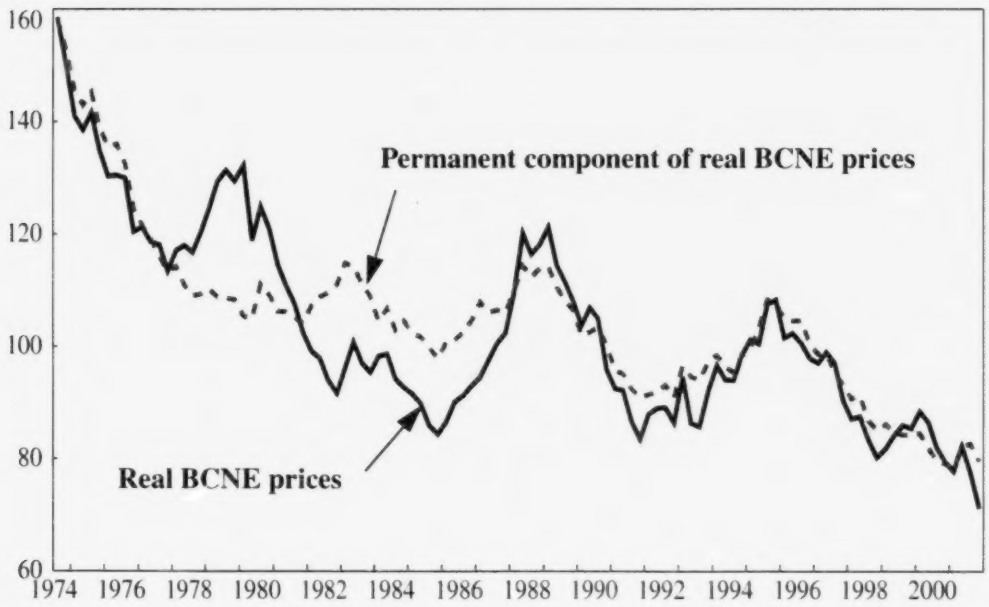


Figure 5: Simulation of a 1 Per Cent Positive Shock to World Output Gap

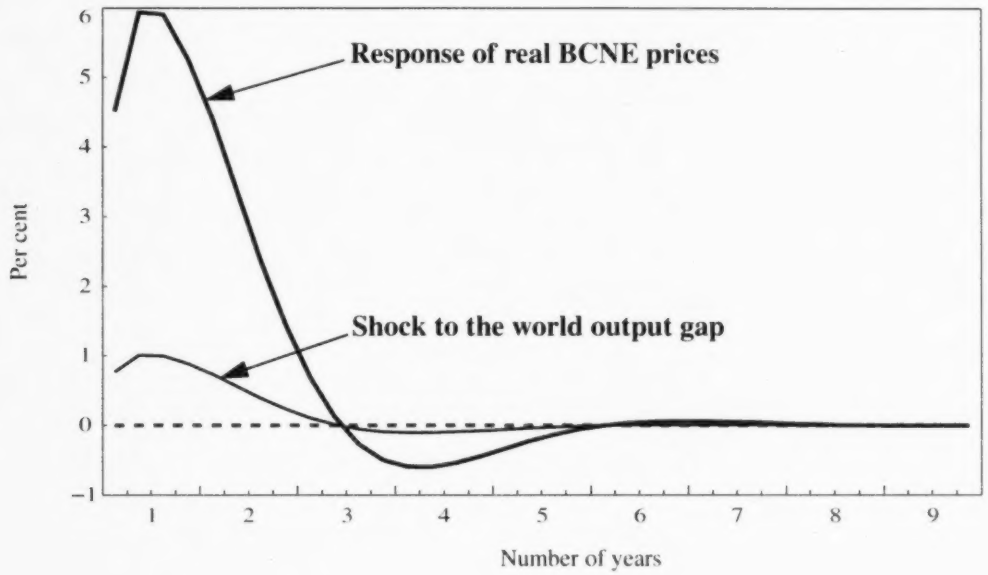


Figure 6: Simulation of a 1 Per Cent Positive Shock to the Real U.S.-Dollar Effective Exchange Rate Gap

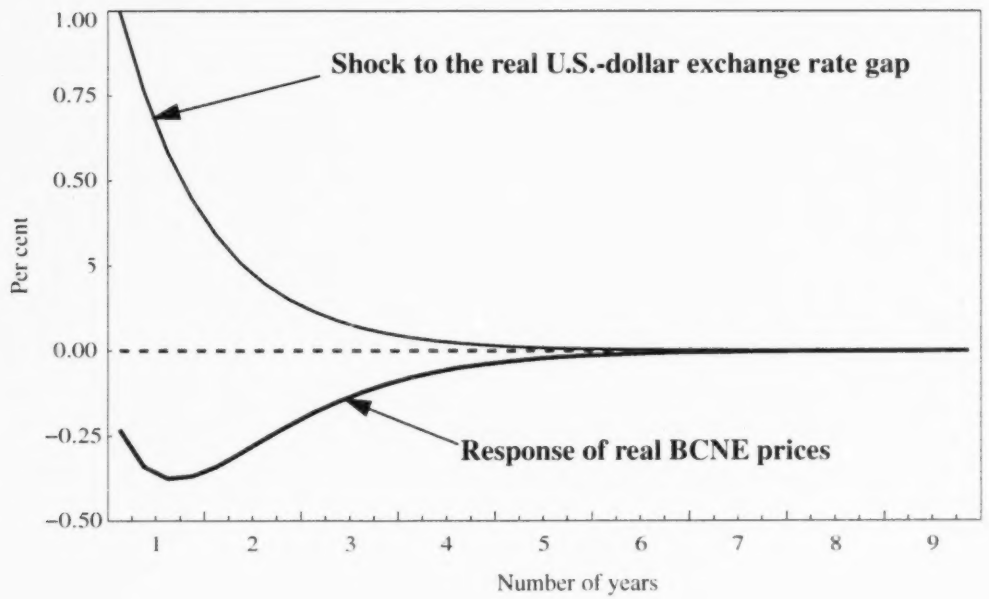


Figure 7: Real Crude Oil Prices

Figure 8: The World Output Gap and the Transitory Component of Real WTI Crude Oil Prices (per cent)

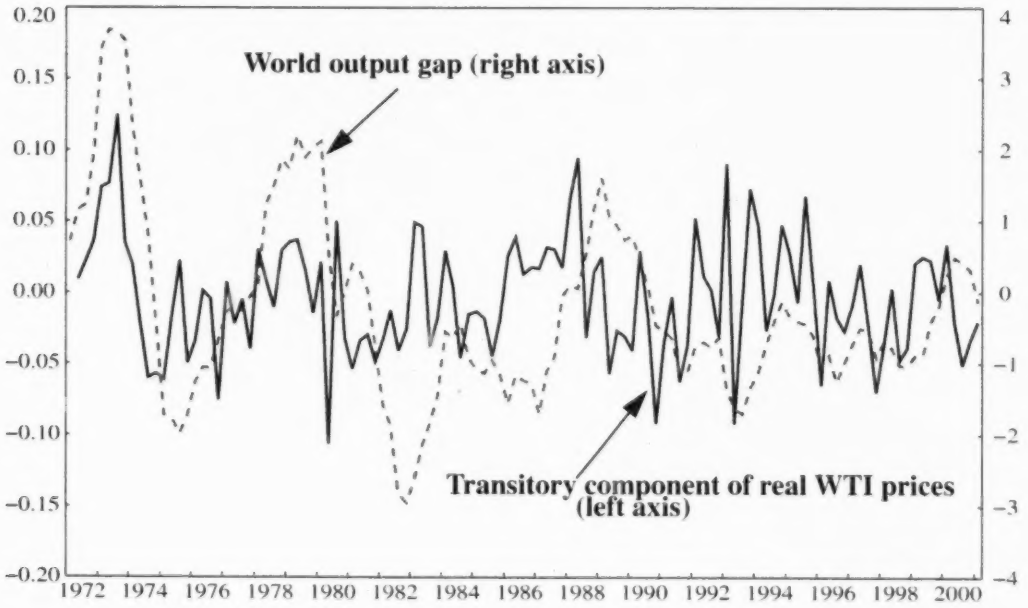
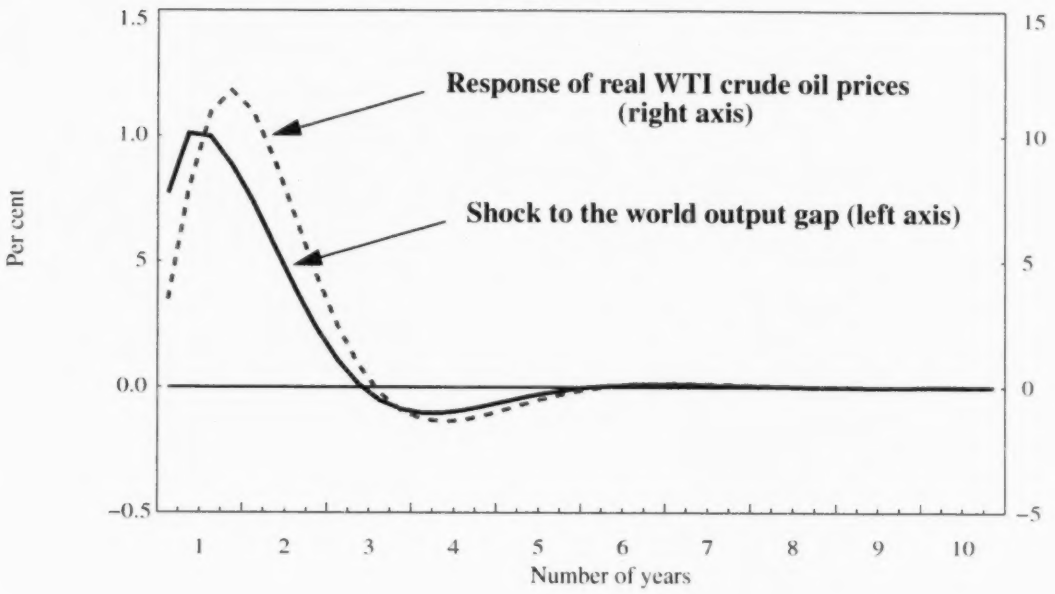


Figure 9: Simulation of a 1 Per Cent Positive Shock to the World Output Gap

(Regime 3)



Appendix A: Weights of Commodities and Subindexes in the BCPI

Item	Weight
Total BCPI	**100.0**
1.0 Energy	**34.9**
Crude oil (WTI)	21.7
Natural gas	10.4
Coal	2.7
2.0 Total BCPI excluding Energy (BCNE)	**65.1**
2.1 Food	18.8
2.1.1 Grains and oilseeds	8.8
Barley	1.2
Canola	1.3
Corn	0.8
Wheat	5.5
2.1.2 Livestock	9.2
Cattle	6.1
Hogs	3.2
2.1.3 Fish	0.7
Cod	0.04
Lobster	0.34
Salmon	0.36
2.2 Industrial materials	46.3
2.2.1 Metals	14.4
Gold	2.8
Silver	0.6
Aluminum	3.0
Copper	2.9
Nickel	2.4
Zinc	2.7
2.2.2 Minerals	2.3
Potash	1.3
Sulphur	1.0
2.2.3 Forest products	29.6
Lumber	9.0
Newsprint	8.3
Pulp	12.3

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Newsprint	8.3
Pulp	12.3

Appendix B: The Blanchard-Quah (1989) Decomposition and the Link Between the Structural Form and the Reduced Form of the Model

The shocks and the variables in the SVAR for real BCNE prices can be defined as follows:

$$\varepsilon_t = \begin{bmatrix} \varepsilon_s \\ \varepsilon_{d1} \\ \varepsilon_{d2} \\ \varepsilon_{d3} \\ \varepsilon_{d4} \end{bmatrix} \text{ and } Z_t = \begin{bmatrix} \Delta Rbcne \\ Wgap \\ \Delta W\pi \\ RRus \\ \Delta Erus \end{bmatrix}, \tag{B1}$$

where ε_s is the only type of shock that will have a permanent effect on real BCNE prices and the other four shocks are restricted to transitory effects on real BCNE prices. Given that we are interested only in the decomposition of real BCNE prices into a permanent component and a *total* transitory component, we treat four transitory shocks as a single aggregate demand-shock term.

The moving-average representation of the structural model is defined as follows:

$$Z_t = \Gamma(0)\varepsilon_t + \Gamma_1\varepsilon_{t-1} + \Gamma_2\varepsilon_{t-2} + \ldots = \Gamma(L)\varepsilon_t, \tag{B2}$$

and the corresponding long-run-effect matrix of the structural shocks is:

$$\Gamma(1) = \Gamma(0) + \Gamma_1 + \Gamma_2 + \ldots + \Gamma_\infty , \tag{B3}$$

where, $E(\varepsilon_t \varepsilon_t) = I$. The diagonal elements are normalized to 1's only for the purpose of simplification.

To identify the structural model, we first estimate the reduced form of the model (i.e., the VAR):

$$Z_t = \sum_{i=1}^{p} \Pi_i Z_{t-i} + e_t, \tag{B4}$$

where p is the number of lags[1] and e_t is the vector of the reduced-form shocks, where

$E(e_t e_t) = \Sigma$.

1. The reduced-form model includes eight lags. We have estimated a model that includes six lags, and the results are almost identical.

Given that the stochastic process is stationary, the moving-average representation of equation (B4) is defined by the following relationship:

$$Z_t = e_t + C_1 e_{t-1} + C_2 e_{t-2} + \ldots = C(L)\, e_t \,, \tag{B5}$$

and the long-run-effect matrix of the reduced-form shocks is:

$$C(1) = 1 + C_1 + C_2 + \ldots + C_\infty \,. \tag{B6}$$

Given equations (B2) and (B5), the reduced-form residuals are linked to the structural residuals in the following way:

$$e_t = \Gamma(0)\varepsilon_t \,. \tag{B7}$$

Consequently,

$$E(e_t\, e_t) = \Gamma(0)\Gamma(0)' \quad \text{because } E(\varepsilon_t\, \varepsilon_t) = I. \tag{B8}$$

In addition, the long-run-effect matrix of the reduced-form shocks, $C(1)$, is linked to the equivalent matrix of the structural shocks ($\Gamma(1)$) and,

$$\Gamma(1) = C(1)\Gamma(0). \tag{B9}$$

To identify the structural model, we need to impose a sufficient number of restrictions on the system of equations formed by equations (B8) and (B9). The fifty elements of the structural-form matrices $\Gamma(0)$ and $\Gamma(1)$ are unknown and the elements of $C(1)$ and $E(e_t e_t)$ are known from the estimation of the reduced-form model. Given that Σ is a symmetric matrix, equations (B8) and (B9) contain forty different relations. Therefore, we have to impose ten restrictions on the elements of $\Gamma(0)$ and $\Gamma(1)$. The Blanchard and Quah decomposition consists of imposing restrictions on the long-run-effect matrix of the structural shocks (i.e., $\Gamma(1)$), instead of imposing a predetermined structure on the variables by the restrictions on the $\Gamma(0)$ matrix. We achieve this by imposing that $\Gamma(1)$ is triangular. Given these restrictions, the system of equations formed by

equations (B8) and (B9) is solvable, and therefore the structural model is identified. Equation (B10) shows the restrictions imposed on the long-run-effect matrix of the structural shocks with, for presentation purposes, the shocks of the structural model on the horizontal axis and the variables of the model, in levels, on the vertical axis:

$$\begin{bmatrix} & \begin{bmatrix} \varepsilon_s & \varepsilon_{d1} & \varepsilon_{d2} & \varepsilon_{d3} & \varepsilon_{d4} \end{bmatrix} \\ \begin{bmatrix} Rbcne \\ \int Wygap \\ W\pi \\ \int RRus \\ Erus \end{bmatrix} & \begin{bmatrix} r_{11} & 0 & 0 & 0 & 0 \\ r_{21} & r_{22} & 0 & 0 & 0 \\ r_{31} & r_{32} & r_{33} & 0 & 0 \\ r_{41} & r_{42} & r_{43} & r_{44} & 0 \\ r_{51} & r_{52} & r_{53} & r_{54} & r_{55} \end{bmatrix} \end{bmatrix} = \Gamma(1). \qquad \text{(B10)}$$

Therefore, we impose that ε_s is the only type of shock that has a permanent effect on real BCNE prices. This gives four restrictions. The other six restrictions are required only to decompose the total transitory component into its four subcomponents. Consequently, they are irrelevant for decomposing real BCNE prices into a permanent and a *total* transitory component. In other words, results concerning the decomposition of real BCNE prices are unaffected by the assumption regarding the ordering of the four last variables. This simply reflects the fact that, in the long run, the model is recursive from top to bottom.

Appendix C: Technical Details on the Bai and Perron (1998) Methodology

We consider a multiple linear regression with m breaks ($m+1$ regimes). The equation of real crude oil prices is specified in a compact matrix notation as:

$$Y = X\beta + \varepsilon,$$

where Y is the observed dependent variable at time t; X is the matrix of covariates, which is partitioned according to the break points T_B; β is the corresponding vector of coefficients; and ε is the disturbance term. The break points $(T_1,...,T_m)$ are explicitly treated as unknown. The purpose is therefore to estimate the regression coefficients and the break points simultaneously when T observations of Y and X are available. The estimation method considered is based on the least-squares principle. For each m-partition $(T1,...,Tm)$, the associated least-squares estimates of β is obtained by minimizing the sum of squared residuals, denoted here as S_T. The estimated coefficients and break points are such that

$$(T_1, \ldots, T_m) = \arg min_{\{T_1,\ldots,T_m\}} S_T T_1, \ldots, T_m),$$

where the minimization is taken over all partitions $(T_1,...,T_m)$, so the break-point estimators are global minimizers of the objective function.

BP proposed a test based on the supremum of the F-statistic, which is called the sup$F$ test, to detect the multiple breaks. This test is labelled as the sup$F(l+1|l)$. The method amounts to the application of $l+1$ tests of the null hypothesis of no structural change versus the alternative hypothesis of l changes. The test is applied to each segment containing the observation T_{m-1} to T_m with $m = 1,...,l+1$. We reject the null hypothesis in favour of a model with $l+1$ breaks if the overall minimal value of squared residuals is sufficiently smaller than the sum of squared residuals from the l-break model. The break date is selected as the one associated with this overall minimum. The asymptotic distribution of the test statistic depends on the selected minimal length of the segments, which is a function of a trimming parameter.[1] To apply the test, we use a trimming of 15 per cent. Hence, given our sample period of 1974–2001, no more than six breaks are allowed, while each regime must have at least sixteen observations.

1. We need to trim the sample by some fraction, since the test statistic diverges to infinity (see Andrews 1993 for details). For this reason, we cannot test for the presence of a structural break in the first/last four years of the sample or the one very close to another break point.

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